*** Dr. G. B. M. Zerr and Mr. J. F. Lawrence prove in general that $a_1+a_2+\dots\dots+a_n>n\sqrt{(a_1a_2\dots\dots a_n)}$. Dr. Zerr also proved that $(a_1{}^m+\dots\dots+a_n{}^m)/n>[(a_1+\dots\dots+a_n)/n]^m$.

210. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

The sum of five quantities and the sum of their cubes are both zero. Show that the sum of their fifth powers is a factor of the sum of any odd powers of the quantities.

Solution by G. W. GREENWOOD, M. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

Denote the quantities by $\alpha$, $\beta$, $\gamma$, $\delta$, $\varepsilon$, and let the equation of which they are the roots be

$$x^5+ax^4+bx^3+cx^2+dx+e=0.$$

Then

$$\Sigma\alpha=-a=0.$$

$$\Sigma\alpha^3=(\Sigma\alpha^2-\Sigma\alpha\beta)\Sigma\alpha-\Sigma\alpha\beta\gamma)=c=0.$$

Substituting the roots in turn for $x$, and adding, we get,

$$\Sigma\alpha^5+5e=0.$$

Multiply the equation by $x^2$, make the same substitutions, and we get in a similar manner,

$$\Sigma\alpha^7+b\Sigma\alpha^5+5e=0,$$

$$i.\ e.,\ \Sigma\alpha^7+(b-1)\Sigma\alpha^5=0.$$

Hence $\Sigma\alpha^5$ is a factor of $\Sigma\alpha^7$ and the process may be repeated indefinitely.

Also solved by Elmer Schuyler, G. B. M. Zerr, J. Scheffer.

## GEOMETRY.

228 Proposed by O.E. GLENN, A.M., Fellow in Mathematics, University of Pennsylvania, Philadelphia, Pa

Given a point $O$ without a circle $S$; two arbitrary lines through $O$ cut $S$ in the points $A$, $A'$, and $B$, $B'$, respectively. Prove, by pure geometry, that the four circles through $OAR$, $OBR$, $OA'R'$, $OB'R'$, respectively, intersect in points collinear with $O$; $R$ and $R'$ being points upon $S$ arbitrarily chosen.

Solution by T. L. CROYES, Paris, France.

Let us take the inverse of the system with regard to $O$, and let the inverses of the five circles $S$, $OAR$, $OBR$, $OA'R'$, $OB'R'$, be a circle $s$, and the four right lines $ar$, $br$, $a'r'$, $b'r'$ ($a$, $a'$, $b$, $b'$, $r$, $r'$ being the inverses of $A$, $A'$, $B$, $B'$, $R$, $R'$).

By Pascal's theorem, the points of intersection $(ar, b'r')(br, a'r')$ are collinear with the point $O$.

237. Proposed by S. A. COREY, Hiteman, Iowa.

Let $AB$, $BC$, $CD$, $DE$, $EA$ be the sides of a pentagon, plain or gauche. Double the length of $CB$ and $DE$ by extending from $B$ and $E$ to $G$ and $H$, re-

spectively. Draw $B'D$ parallel to and of the same currency as $BC$. Connect $G$ and $H$. Then prove that $2(AB^2+BC^2+CD^2+DE^2+EA^2)=3CD^2+4(DE.BC.\cos EDB'+EA.AB.\cos EAB)+GH^2$.

Solution by the PROPOSER.

Let $a$, $b$, $c$, and $d$ be the vector sides corresponding with $DE$, $EA$, $AB$, and $BC$, respectively. Then will $-(a+b+c+d)$ represent the side $CD$, and $(a-b-c+d)$, the line $GH$. Squaring each of these six vectors to get their squared tensors, and adding as follows:

$$2[T^2a+T^2b+T^2c+T^2d+T^2.(-a-b-c-d)]$$
$$-3T^2.(-a-b-c-d)-T^2.(a-b-c+d)$$

is found to be $4(Ta.Td.\cos ad+Tb.Tc.\cos bc)$ as required by the problem.*

238. Proposed by O. W. ANTHONY, Head of the Mathematical Department, DeWitt Clinton High School, New York.

Construct a trapezoid having given the sum of the parallel sides, the sum of the diagonals, and the angle formed by the diagonals.

I. Solution by J. H. MEYERS, S. J., Professsor of Mathematics in Spring Hill College, Mobile, Ala.

Construct the triangle $ABC$, whose base $AB$=sum of parallel sides, angle $C$=angle between diagonals and where $AC+CB$=sum of diagonals. [See Casey's *Sequel to Euclid*, Book III, Prop. 29.]†

Take the point $E$ on $AB$, such that $CE=CB$; from $E$ draw $EF$ parallel and equal to $AC$ meeting $CB$ in $O$; join $B$ and $F$. $CFBE$ is the required trapezoid. In proof, $AE=CF$, hence $EB+CF$=given sum of parallel sides. Since $EF=AC$, $EF+CB$=given sum of parallel sides. And finally, angle $EOB=ACB$.

Analogously solved by J. Scheffer.

II. Remark by G. W. GREENWOOD, M. A., Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

Consider any trapezoid $ABCD$ satisfying the given conditions, where $AB$, $CD$ are the parallel sides. Draw any parallel to $AC$ meeting $AB$ internally in $A'$, say, and hence meeting $CD$ internally in $C'$. The trapezoid $A'BC'D$ can easily be seen to fulfill the required conditions. Therefore the solution is not unique.

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*In the above solution the lines $CB$ and $DE$ are the ones produced. Prof. G. W. Greenwood has demonstrated that the corresponding theorem, where the lines produced are $BC$ and $ED$, does not hold true. ED. E.

†Describe on the same side of $AB$ as a chord the arcs $AXB$, $AYB$ such that angle $AXB$=given angle $\alpha$ for every point $X$, and $AYB=\frac{1}{2}\alpha$. Determine the point $P$ on arc $AYB$ such that $AP$=sum of diagonals of trapezoid (*i. e.*, sum of sides of the required triangle). Let $AP$ meet arc $AXB$ in $C$. Then since angle $CPB+CBP=\alpha$, angle $CBP=\frac{1}{2}\alpha$ and $CP=CB$. Therefore $ABC$ is the required triangle. ED. E.